1. BROJEVI

1.1. Brojevni sustavi

1.1. $221_{(5)} = 2 \cdot 5^2 + 2 \cdot 5^1 + 1 = 50 + 10 + 1 = 61$

$110011_{(2)} = 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 = 32 + 16 + 2 + 1 = 51$

$567_{(8)} = 5 \cdot 8^2 + 6 \cdot 8^1 + 7 = 375$

$21000_{(3)} = 2 \cdot 3^4 + 1 \cdot 3^3 = 189$

$5550_{(6)} = 5 \cdot 6^3 + 5 \cdot 6^2 + 5 \cdot 6^1 = 1080 + 180 + 30 = 1290$

1.2. 1) $1011_{(2)} = 1 \cdot 2^3 + 1 \cdot 2^1 + 1 = 11$

2) $100101_{(2)} = 1 \cdot 2^5 + 1 \cdot 2^2 + 1 = 32 + 4 + 1 = 37$

3) $1100101_{(2)} = 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^2 + 1 = 64 + 32 + 4 + 1 = 101$

4) $110001011_{(2)} = 1 \cdot 2^8 + 1 \cdot 2^7 + 1 \cdot 2^3 + 1 \cdot 2^1 + 1 = 395$

5) $11_{(8)} = 1 \cdot 8^1 + 1 = 9$

6) $24_{(8)} = 2 \cdot 8^1 + 4 = 20$

7) $126_{(8)} = 1 \cdot 8^2 + 2 \cdot 8^1 + 6 = 64 + 16 + 6 = 86$

8) $3201_{(8)} = 3 \cdot 8^3 + 2 \cdot 8^2 + 1 = 1536 + 128 + 1 = 1665$

9) $8_{(16)} = 8$

10) $20_{(16)} = 2 \cdot 16 + 0 = 32$

11) $3A_{(16)} = 3 \cdot 16 + 10 = 58$ (U sustavu s bazom 16 je A = 10, B = 11, ..., F = 15)

12) $2EB1_{(16)} = 2 \cdot 16^3 + 14 \cdot 16^2 + 11 \cdot 16^1 + 1 = 8192 + 3584 + 176 + 1 = 11953$

1.3. Poznato je da jedna znamenka oktalnog (heksadekadskog) sustava odgovora točno trima (četirima) znamenkama binarnog sustava.

Heksadekadski	Binarni
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Oktalni	Binarni
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111

Koristeći ove tablice višeznamenkasti broj zapisan u binarnom sustavu lako možemo prevesti u oktalni ili heksadekadski sustav i obratno. Zapišimo najprije zadane brojeve u binarnom sustavu (metodom dijeljenja).

a) 6 : 2 = 3 ostatak 0 ↑
3 : 2 = 1 ostatak 1 ↑
1 : 2 = 0 ostatak 1

$6 = \overbrace{110}^{6}{}_{(2)} = 6_{(8)} = 6_{(16)}$

b) 13 : 2 = 6 ostatak 1 ↑
6 : 2 = 3 ostatak 0 ↑
3 : 2 = 1 ostatak 1 ↑
1 : 2 = 0 ostatak 1

$13 = \overbrace{1}^{1}\ \overbrace{101}^{5}{}_{(2)} = 15_{(8)} = D_{(16)}$

c) 25 : 2 = 12 ostatak 1 ↑
12 : 2 = 6 ostatak 0 ↑
6 : 2 = 3 ostatak 0 ↑
3 : 2 = 1 ostatak 1 ↑
1 : 2 = 0 ostatak 1

$25 = \overbrace{11}^{3}\ \overbrace{001}^{1}{}_{(2)} = 31_{(8)} = 19_{(16)}$

d) Sami dijelite.

$125 = \overbrace{111}^{1}\ \overbrace{101}^{7}\ \overbrace{}^{5}{}_{(2)} = 175_{(8)} = 7D_{(16)}$

1.4. a) 6 : 5 = 1 ost. 1 ↑
1 : 5 = 0 ost. 1 ↑

6 : 9 = 0 ost. 6 ↑
6 : 12 = 0 ost. 6 ↑

$6 = 11_{(5)} = 6_{(9)} = 6_{(12)}$

b) 13 : 5 = 2 ost. 3 ↑ 13 : 9 = 1 ost. 4 ↑ 13 : 12 = 1 ost. 1 ↑
2 : 5 = 0 ost. 2 ↑ 1 : 9 = 0 ost. 1 ↑ 1 : 12 = 0 ost. 1 ↑
$13 = 23_{(5)} = 14_{(9)} = 11_{(12)}$

c) 25 : 5 = 5 ost. 0 ↑ 25 : 9 = 2 ost. 7 ↑ 25 : 12 = 2 ost. 1 ↑
5 : 5 = 1 ost. 0 ↑ 2 : 9 = 0 ost. 2 ↑ 2 : 12 = 0 ost. 2 ↑
1 : 5 = 0 ost. 1 ↑ $25 = 100_{(5)} = 27_{(9)} = 21_{(12)}$

d) 125 : 5 = 25 ost. 0 ↑ 125 : 9 = 13 ost. 8 ↑ 125 : 12 = 10 ost. 5 ↑
25 : 5 = 5 ost. 0 ↑ 13 : 9 = 1 ost. 4 ↑ 10 : 12 = 0 ost. 10 = A ↑
5 : 5 = 1 ost. 0 ↑ 1 : 9 = 0 ost. 1 ↑
1 : 5 = 0 ost. 1 ↑ $125 = 1000_{(5)} = 148_{(9)} = A5_{(12)}$

1.5. a) U bazi 2:

11 : 2 = 5 ost. 1 ↑ 100 : 2 = 50 ost. 0 ↑ 222 : 2 = 111 ost. 0 ↑ 1001 : 2 = 500 ost. 1 ↑
5 : 2 = 2 ost. 1 ↑ 50 : 2 = 25 ost. 0 ↑ 111 : 2 = 55 ost. 1 ↑ 500 : 2 = 250 ost. 0 ↑
2 : 2 = 1 ost. 0 ↑ 25 : 2 = 12 ost. 1 ↑ 55 : 2 = 27 ost. 1 ↑ 250 : 2 = 125 ost. 0 ↑
1 : 2 = 0 ost. 1 ↑ 12 : 2 = 6 ost. 0 ↑ 27 : 2 = 13 ost. 1 ↑ 125 : 2 = 62 ost. 1 ↑
$11 = 1011_{(2)}$ 6 : 2 = 3 ost. 0 ↑ 13 : 2 = 6 ost. 1 ↑ 62 : 2 = 31 ost. 0 ↑
33 : 2 = 16 ost. 1 ↑ 3 : 2 = 1 ost. 1 ↑ 6 : 2 = 3 ost. 0 ↑ 31 : 2 = 15 ost. 1 ↑
16 : 2 = 8 ost. 0 ↑ 1 : 2 = 0 ost. 1 ↑ 3 : 2 = 1 ost. 1 ↑ 15 : 2 = 7 ost. 1 ↑
8 : 2 = 4 ost. 0 ↑ $100 = 1100100_{(2)}$ 1 : 2 = 0 ost. 1 ↑ 7 : 2 = 3 ost. 1 ↑
4 : 2 = 2 ost. 0 ↑ $222 = 11011110_{(2)}$ 3 : 2 = 1 ost. 1 ↑
2 : 2 = 1 ost. 0 ↑ 1 : 2 = 0 ost. 1 ↑
1 : 2 = 0 ost. 1 ↑ $1001 = 1111101001_{(2)}$
$33 = 100001_{(2)}$

b) U bazi 5:

11 : 5 = 2 ost. 1 ↑ 222 : 5 = 44 ost. 2 ↑
2 : 5 = 0 ost. 2 ↑ 44 : 5 = 8 ost. 4 ↑
$11 = 21_{(5)}$ 8 : 5 = 1 ost. 3 ↑
33 : 5 = 6 ost. 3 ↑ 1 : 5 = 0 ost. 1 ↑
6 : 5 = 1 ost. 1 ↑ $222 = 1342_{(5)}$
1 : 5 = 0 ost. 1 ↑ 1001 : 5 = 200 ost. 1 ↑
$33 = 113_{(5)}$ 200 : 5 = 40 ost. 0 ↑
100 : 5 = 20 ost. 0 ↑ 40 : 5 = 8 ost. 0 ↑
20 : 5 = 4 ost. 0 ↑ 8 : 5 = 1 ost. 3 ↑
4 : 5 = 0 ost. 4 ↑ 1 : 5 = 0 ost. 1 ↑
$100 = 400_{(5)}$ $1001 = 13001_{(5)}$

c) U bazi 12:

11 : 12 = 0 ost. 11 = B ↑
$11 = B_{(12)}$
33 : 12 = 2 ost. 9 ↑
2 : 12 = 0 ost. 2 ↑
$33 = 29_{(12)}$
100 : 12 = 8 ost. 4 ↑
8 : 12 = 0 ost. 8 ↑
$100 = 84_{(12)}$
222 : 12 = 18 ost. 6 ↑
18 : 12 = 1 ost. 6 ↑
1 : 12 = 0 ost. 1 ↑
$222 = 166_{(12)}$
1001 : 12 = 83 ost. 5
83 : 12 = 6 ost. 11 = B ↑
6 : 12 = 0 ost. 6 ↑
$1001 = 6B5_{(12)}$

1.6. Vidite tablice uz zadatak 1.3.

$\overbrace{1\,011}^{1\ 3}$ = $13_{(8)}$ = $B_{(16)}$ (B)

$\overbrace{11\,000\,110}^{3\ 0\ 6}$ = $306_{(8)}$ = $C6_{(16)}$ (C 6)

$\overbrace{11\,001\,100\,101\,110\,011}^{3\ 1\ 4\ 5\ 6\ 3}$ = (1 9 9 7 3)

$= 314563_{(8)} = 19973_{(16)}$

1.7. Vidite zadatak 1.3. Najprije ćemo zadane brojeve iz heksadekadskog sustava prebaciti u binarni sustav, a zatim u oktalni sustav.

$58_{(16)} = \overbrace{101\,1000}^{5\ 8}{}_{(2)} = 130_{(8)}$ (1 3 0)

$1A2_{(16)} = \overbrace{1\,1010\,0010}^{1\ A\ 2}{}_{(2)} = 642_{(8)}$ (6 4 2)

$FFFF_{(16)} = \overbrace{1111}^{F}\,\overbrace{1111}^{F}\,\overbrace{1111}^{F}\,\overbrace{1111}^{F}{}_{(2)} = 177777_{(8)}$

1.8.

$223_{(8)} = \overbrace{10}^{2}\,\overbrace{010}^{2}\,\overbrace{011}^{3}{}_{(2)} = 93_{(16)}$ $\qquad$ $517_{(8)} = \overbrace{101}^{5}\,\overbrace{001}^{1}\,\overbrace{111}^{7}{}_{(2)} = 14F_{(16)}$

$12053_{(8)} = \overbrace{1}^{1}\,\overbrace{010}^{2}\,\overbrace{000}^{0}\,\overbrace{101}^{5}\,\overbrace{011}^{3}{}_{(2)} = 142B_{(16)}$

1.9. $212_{(3)} = 2 \cdot 3^2 + 1 \cdot 3^1 + 2 =$
$= 23_{(10)} = 10111_{(2)}$

23 : 2 = 11 ost. 1
11 : 2 = 5 ost. 1
5 : 2 = 2 ost. 1
2 : 2 = 1 ost. 0
1 : 2 = 0 ost. 1

$30_{(4)} = 3 \cdot 4^1 + 0 =$
$= 12 = 1100_{(2)}$

12 : 2 = 6 ost. 0
6 : 2 = 3 ost. 0
3 : 2 = 1 ost. 1
1 : 2 = 0 ost. 1

$245_{(6)} = 2 \cdot 6^2 + 4 \cdot 6^1 + 5 =$
$= 72 + 24 + 5 = 101 = 1100101_{(2)}$

101 : 2 = 50 ost. 1
50 : 2 = 25 ost. 0
25 : 2 = 12 ost. 1
12 : 2 = 6 ost. 0
6 : 2 = 3 ost. 0
3 : 2 = 1 ost. 1
1 : 2 = 0 ost. 1

$177_{(8)} = 127 = 1111111_{(2)}$

$28_{(12)} = 30 = 11110_{(2)}$

1.10.

a) $\overbrace{101}^{5}\,\overbrace{110}^{6}{}_{(2)} = 56_{(8)}$

b) $2102_{(3)} = 2 \cdot 3^3 + 1 \cdot 3^2 + 2 =$
$= 65 = 101_{(8)}$

65 : 8 = 8 ost. 1
8 : 8 = 1 ost. 0
1 : 8 = 0 ost. 1

c) $3220_{(4)} = 3 \cdot 4^3 + 2 \cdot 4^2 + 2 \cdot 4^1 + 0 = 232 = 350_{(8)}$

d) $11011_{(5)} = 756 = 1364_{(8)}$

756 : 8 = 94 ost. 4
94 : 8 = 11 ost. 6
11 : 8 = 1 ost. 3
1 : 8 = 0 ost. 1

1.11. 1) 10, 11, 12, 20, 21, …
22, 100, 101, 102, 110, …

Koriste se znamenke {0, 1, 2} pa naslućujemo da je niz napisan u sustavu s bazom 3.

$10_{(3)} = 1 \cdot 3^1 + 0 = 3$
$11_{(3)} = 1 \cdot 3^1 + 1 = 4$
$12_{(3)} = 1 \cdot 3^1 + 2 = 5$
$20_{(3)} = 2 \cdot 3^1 + 0 = 6$
$21_{(3)} = 2 \cdot 3^1 + 1 = 7$

$22_{(3)} = 2 \cdot 3^1 + 2 = 8$
$100 = 1 \cdot 3^2 = 9$
$101 = 1 \cdot 3^2 + 1 = 10$
$102 = 1 \cdot 3^2 + 2 = 11$
$110 = 1 \cdot 3^2 + 3 = 12$

2) 101, 110, 111, 1000, 1001, …
1010, 1011, 1100, 1101, 1110, …

$101_{(2)} = 1 \cdot 2^2 + 0 \cdot 2^1 + 1 = 5$
$110_{(2)} = 1 \cdot 2^2 + 1 \cdot 2^1 + 0 = 6$
$111_{(2)} = 1 \cdot 2^2 + 1 \cdot 2^1 + 1 = 7$
$1000_{(2)} = 1 \cdot 2^3 = 8$
$1001_{(2)} = 1 \cdot 2^3 + 1 = 9$

$10 = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 = 1010$
$11 = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 = 1011$
$12 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 = 1100$
$13 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 = 1101$
$14 = 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 = 1110$

3) 1, 2, 3, 10, 11, 12, …
13, 20, 21, 22, 23, … Niz je napisan u sustavu s bazom 4.

$1_{(4)} = 1$
$2_{(4)} = 2$
$3_{(4)} = 3$
$10_{(4)} = 1 \cdot 4 + 0 = 4$
$11_{(4)} = 1 \cdot 4 + 1 = 5$

$12_{(4)} = 1 \cdot 4 + 2 = 6$
$13_{(4)} = 1 \cdot 4 + 3 = 7$
$20_{(4)} = 2 \cdot 4 + 0 = 8$
$21_{(4)} = 2 \cdot 4 + 1 = 9$
$22_{(4)} = 2 \cdot 4 + 2 = 10$
$23_{(4)} = 2 \cdot 4 + 3 = 11$

4) 23, 24, 25, 30, 31, 32, …

33, 34, 35, 40, 41, … Niz je napisan u sustavu s bazom 6.

$23_{(6)} = 2 \cdot 6^1 + 3 = 15$ $\quad 21 = 3 \cdot 6^1 + 3 = 33_{(6)}$

$24_{(6)} = 2 \cdot 6^1 + 4 = 16$ $\quad 22 = 3 \cdot 6^1 + 4 = 34_{(6)}$

$25_{(6)} = 2 \cdot 6^1 + 5 = 17$ $\quad 23 = 3 \cdot 6^1 + 5 = 35_{(6)}$

$30_{(6)} = 3 \cdot 6^1 + 0 = 18$ $\quad 24 = 4 \cdot 6^1 + 0 = 40_{(6)}$

$31_{(6)} = 3 \cdot 6^1 + 1 = 19$ $\quad 25 = 4 \cdot 6^1 + 1 = 41_{(6)}$

$32_{(6)} = 3 \cdot 6^1 + 2 = 20$

1.12. 1) 10011, 10010, 10001, 10000, … Niz je napisan u sustavu

1111, 1110, 1101, 1100, … s bazom 2.

$10011_{(2)} = 1 \cdot 2^4 + 1 \cdot 2^1 + 1 = 19$ $\quad 15 = 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 = 1111_{(2)}$

$10010_{(2)} = 1 \cdot 2^4 + 1 \cdot 2^1 + 0 = 18$ $\quad 14 = 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 = 1110_{(2)}$

$10001_{(2)} = 1 \cdot 2^4 + 1 = 17$ $\quad 13 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 = 1101_{(2)}$

$10000_{(2)} = 1 \cdot 2^4 = 16$ $\quad 12 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 = 1100_{(2)}$

2) 10, 100, 110, 1000, 1010, …

1100, 1110, 10000, 10010, …

$10_{(2)} = 1 \cdot 2^1 + 0 = 2$ $\quad 12 = 1 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 = 1100_{(2)}$

$100_{(2)} = 1 \cdot 2^2 = 4$ $\quad 14 = 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 = 1110_{(2)}$

$110_{(2)} = 1 \cdot 2^2 + 1 \cdot 2^1 + 0 = 6$ $\quad 16 = 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 = 10000_{(2)}$

$1000_{(2)} = 1 \cdot 2^3 = 8$ $\quad 18 = 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 = 10010_{(2)}$

$1010_{(2)} = 1 \cdot 2^3 + 1 \cdot 2^1 = 10$

3) 10, 20, 100, 110, 120, …

200, 210, 220, 1000, … Niz je zapisan u sustavu s bazom 3.

$10_{(3)} = 1 \cdot 3^1 + 0 = 3$ $\quad 18 = 2 \cdot 3^2 + 0 \cdot 3^1 + 0 = 200_{(3)}$

$20_{(3)} = 2 \cdot 3^1 + 0 = 6$ $\quad 21 = 2 \cdot 3^2 + 1 \cdot 3^1 + 0 = 210_{(3)}$

$100_{(3)} = 1 \cdot 3^2 = 9$ $\quad 24 = 2 \cdot 3^2 + 2 \cdot 3^1 + 0 = 220_{(3)}$

$110_{(3)} = 1 \cdot 3^2 + 1 \cdot 3^1 = 12$ $\quad 27 = 1 \cdot 3^3 + 0 \cdot 3^2 + 0 \cdot 3^1 + 0 = 1000_{(3)}$

$120_{(3)} = 1 \cdot 3^2 + 2 \cdot 3^1 = 15$

4) 11, 13, 20, 22,24, …

31, 33, 40, 42,44, … Niz je napisan u sustavu s bazom 5.

$11_{(5)} = 1 \cdot 5^1 + 1 = 6$ $\quad 16 = 3 \cdot 5^1 + 1 = 31_{(5)}$

$13_{(5)} = 1 \cdot 5^1 + 3 = 8$ $\quad 18 = 3 \cdot 5^1 + 3 = 33_{(5)}$

$20_{(5)} = 2 \cdot 5^1 + 0 = 10$ $\quad 20 = 4 \cdot 5^1 + 0 = 40_{(5)}$

$22_{(5)} = 2 \cdot 5^1 + 2 = 12$ $\quad 22 = 4 \cdot 5^1 + 2 = 42_{(5)}$

$24_{(5)} = 2 \cdot 5^1 + 4 = 14$ $\quad 24 = 4 \cdot 5^1 + 4 = 44_{(5)}$

1.13. 1) $101_{(x)} + 1001_{(x)} = 1110_{(x)}$ $\quad \cancel{1 \cdot x^2} + 0 \cdot x + 1 + \cancel{1 \cdot x^3} + 1 = \cancel{1 \cdot x^3} + \cancel{1 \cdot x^2} + 1 \cdot x$

$x = 2$

2) $1211_{(x)} - 1120_{(x)} = 21_{(x)}$ $\quad (1 \cdot x^3 + 2 \cdot x^2 + 1 \cdot x + 1) - (1 \cdot x^3 + 1 \cdot x^2 + 2 \cdot x) = 2x + 1$

$x^2 - 3x = 0 \quad \Rightarrow \quad \cancel{x_1 = 0}, \quad x_2 = 3$

1.14. 1) $10101 + 1101 = 11202$ U svakom sustavu osim u binarnom.

2) $1211 - 1011 = 200$ U svakom sustavu osim u binarnom.

1.15. Prebacimo zadani broj u dekadski sustav.

$121_{(5)} = 1 \cdot 5^2 + 2 \cdot 5^1 + 1 = 36$

$x + (x + 1) + (x + 2) = 36$

$\underline{x = 11}$

$x + 1 = 12$ } Traženi brojevi

$x + 2 = 13$ } u dekadskom sustavu.

$11 : 5 = 2$ ost. 1 ↑

$2 : 5 = 0$ ost. 2 ↑

$11 = 21_{(5)}$

$12 : 5 = 2$ ost. 2 ↑

$2 : 5 = 0$ ost. 2 ↑

$12 = 22_{(5)}$

$13 : 5 = 2$ ost. 3 ↑

$2 : 5 = 0$ ost. 2 ↑

$13 = 23_{(5)}$

Traženi brojevi su: $21_{(5)}$, $22_{(5)}$ i $23_{(5)}$.

1.16. Postupamo kao prethodnom primjeru.

$11110_{(2)} = 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 16 + 8 + 4 + 2 = 30$

$(2k - 2) + 2k + (2k + 2) = 30 \quad \Rightarrow \quad k = 5$

U dekadskom sustavu to su brojevi 8, 10 i 12; a u binarnom $1000_{(2)}$, $1010_{(2)}$ i $1100_{(2)}$.

1. 17. $440_{(5)} = 4 \cdot 5^2 + 4 \cdot 5^1 = 120$

$(n - 1) \cdot n \cdot (n + 1) = 120$

$n^3 - n - 120 = 0$

$(n^3 - 125) - (n - 5) = 0$

$(n - 5)(n^2 + 5n + 25 - 1) = 0$

$(n - 5)\underbrace{(n^2 + 5n + 24)}_{\neq 0} = 0$

$n - 5 = 0$

$\underline{n = 5}$

U dekadskom sustavu to su brojevi 4, 5 i 6; a u sustavu s bazom 5: $4_{(5)}$, $10_{(5)}$ i $11_{(5)}$.

1.18. $1320_{(9)} = 1 \cdot 9^3 + 3 \cdot 9^2 + 2 \cdot 9^1 = 990$

$(n - 1) \cdot n \cdot (n + 1) = 990$

$n^3 - n - 990 = 0$

$(n^3 - 1000) - (n - 10) = 0$

$(n - 10)\underbrace{(n^2 + 10n + 100 - 1)}_{\neq 0} = 0$

$n - 10 = 0$

$n = 10$

U dekadskom sustavu to su brojevi 9, 10, 11; a u sustavu s bazom 9 to su brojevi $10_{(9)}$, $11_{(9)}$ i $12_{(9)}$.

1.19. $100\,011_{(2)} = 1 \cdot 2^5 + 1 \cdot 2^1 + 1 = 35$

$(2k - 1)(2k + 1) = 35 \quad \Rightarrow \quad k = 3$

Radi se o brojevima 5 i 7, tj. $101_{(2)}$ i $111_{(2)}$.

[Pređite u dekadski.]

1.20. 1) Prema zadatku je: $\frac{1}{3} \cdot 41_{(x)} = 12_{(x)}$

$41_{(x)} = 3 \cdot 12_{(x)}$

$4 \cdot x + 1 = 3 \cdot (1 \cdot x + 2) \quad \Rightarrow x = 5.$

Sada je: $103_{(5)} : 4_{(5)} = 28 : 4 = 7 = 12_{(5)}$.

2) $\frac{3}{4} \cdot 20_{(x)} = 16_{(x)}$

$3 \cdot (2 \cdot x^1 + 0) = 4 \cdot (1 \cdot x^1 + 6) \Rightarrow x = 12$

$\frac{1}{5} \cdot 10_{(12)} = \frac{1}{5} \cdot (1 \cdot 12 + 0) = \frac{12}{5} = 2\frac{2}{5}$

1.21. 1) $23_{(x)} = 41_{(y)}$

$2 \cdot x + 3 = 4 \cdot y + 1$

$2x = 4y - 2 \ /:2$

$x = 2y - 1$

Zaključujemo da je $y > 4$ (koristi znamenke 1 i 4).

Dakle, $(x,y) = (2y - 1, y > 4)$, tj. $\forall y > 4$ imamo uređen par kao rješenje; tj. imamo beskonačno rješenja.

Npr. za $y = 5$, je $x = 9$ tj. $23_{(9)} = 41_{(5)}$.

2) $144_{(x)} = 100_{(y)}$

$1 \cdot x^2 + 4 \cdot x + 4 = 1 \cdot y^2$

$(x + 2)^2 = y^2 \quad \Rightarrow \quad x + 2 = y$

Očito je $x > 4$. Imamo beskonačno rješenja $(x > 4, x + 2)$.

1.22. 1) $ab_{(5)} = ba_{(7)} \quad \Rightarrow \quad a, b \in \{0, 1, 2, 3, 4\}$

$a \cdot 5 + b = b \cdot 7 + a$

$4a = 6b$

$a = \frac{3}{2}b$

Za $b = 4$, $a = \frac{3}{2} \cdot 4 = 6$, ne može biti.

Za $b = 2$, $a = 3$. Dakle, $32_{(5)} = 23_{(7)}$.

2) $abc_{(5)} = cba_{(8)} \quad \Rightarrow \quad a, b, c \in \{0, 1, 2, 3, 4\}$

$a \cdot 5^2 + b \cdot 5 + c = c \cdot 8^2 + b \cdot 8 + a$

$24a - 63c = 3b$

$b = 8a - 21c$

Za $a = 3, c = 1 \quad \Rightarrow \quad b = 3;$

tj. $331_{(5)} = 133_{(8)}$.

3) $aba_{(4)} = bab_{(6)} \quad \Rightarrow \quad a, b \in \{0, 1, 2, 3\}$

$a \cdot 4^2 + b \cdot 4 + a = b \cdot 6^2 + a \cdot 6 + b$

$a = 3b$

Za $b = 1 \Rightarrow a = 3,$

tj. $313_{(4)} = 131_{(6)}$.

1. 23. 1) $101_{(x)} \cdot 11_{(x)} = 1111_{(x)}$

$(x^2 + 1)(x + 1) = x^3 + x^2 + x + 1$

U svakom jer se radi o identitetu za $\forall x \in N$.

2) $1001_{(x)} \cdot 111_{(x)} = 111\ 111_{(x)}$

$(x^3 + 1)(x^2 + x + 1) = x^5 + x^4 + x^3 + x^2 + x + 1$

$x^5 + x^4 + x^3 + x^2 + x + 1 = x^5 + x^4 + x^3 + x^2 + x + 1$

U svakom jer za $\forall x \in N$ imamo identitet. Općenito vrijedi:

$(x^n + 1)(x^{n-1} + x^{n-2} + \ldots + x + 1) = x^{2n-1} + x^{2n-2} + \ldots + x + 1.$

1.24. $11_{(x)} \cdot 12_{(x)} \cdot 13_{(x)} = 3102_{(x)}$

$(x + 1)(x + 2)(x + 3) = 3x^3 + x^2 + 2$

$2x^3 - 5x^2 - 11x - 4 = 0$

$\underbrace{2x^3 - 8x^2} + \underbrace{3x^2 - 12x} + \underbrace{x - 4} = 0$

$x - 4 = 0 \quad x_1 = 4$

$2x^2 + 3x + 1 = 0 \quad x_2 = -1,\ x_3 = -\frac{1}{2}$

$2x^2(x - 4) + 3x(x - 4) + (x - 4) = 0$

$(x - 4)(2x^2 + 3x + 1) = 0$

U sustavu s bazom 4 (to je jedino prirodno rješenje).

1.25. $144_{(n)} = n^2 + 4n + 4 = (n + 2)^2$

U svakom sustavu s bazom n, $n > 4$.

1. 26. $[125_{(x)}]^2 = 16\ 324_{(x)}$

$(x^2 + 2x + 5)^2 = x^4 + 6x^3 + 3x^2 + 2x + 4$

$x^4 + 4x^2 + 25 + 4x^3 + 10x^2 + 20x = x^4 + 6x^3 + 3x^2 + 2x + 4$

$2x^3 - 11x^2 - 18x - 21 = 0$

$\underline{2x^3} + \underline{3x^2} - 14x^2 + \underline{3x} - 21x - 21 = 0$

$x(2x^2 + 3x + 3) - 7(2x^2 + 3x + 3) = 0$

$\underbrace{(2x^2 + 3x + 3)}_{\neq 0}(x - 7) = 0$

$x = 7$

U sustavu s bazom 7.

1. 27. $[24_{(x)}]^2 = 620_{(x)}$

$(2x + 4)^2 = 6 \cdot x^2 + 2x$

$x^2 - 7x - 8 = 0 \quad \Rightarrow \quad x = 8$

U sustavu s bazom 8.

1. 28. $[21_{(x)}]^3 = 20\ 311_{(x)}$

$(2x + 1)^3 = 2x^4 + 3x^2 + x + 1$

$2x^4 - 8x^3 - 9x^2 - 5x = 0$

$x(2x^3 - 8x^2 - 9x - 5) = 0$

$\underline{2x^3} - 10x^2 + \underline{2x^2} - 10x + \underline{x} - 5 = 0$

$x(2x^2 + 2x + 1) - 5(2x^2 + 2x + 1) = 0$

$(x - 5)\underbrace{(2x^2 + 2x + 1)}_{\neq 0} = 0 \quad x = 5$

U sustavu s bazom 5.

1. 29. U svakom sustavu s bazom n, $n > 3$, jer je $1331_{(n)} = n^3 + 3n^2 + 3n + 1 = (n + 1)^3$. Baza mora biti veća od 3 jer se u zapisu broja pojavljuje znamenka 3.

1. 30. 1) $1101_{(2)} + 10101_{(2)} = 100010_{(2)}$

2) $10101_{(2)} + 10111_{(2)} = 101100_{(2)}$

3) $110111_{(2)} + 10101101_{(2)} = 11100100_{(2)}$

4) $1100101_{(2)} + 11001011_{(2)} = 100110000_{(2)}$

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1. 31. 1) $1111_{(2)} - 1001_{(2)} = 110_{(2)}$

2) $11001_{(2)} - 1101_{(2)} = 1100_{(2)}$

3) $110011_{(2)} - 10110_{(2)} = 11101_{(2)}$

4) $110010111_{(2)} - 1101011_{(2)} = 100101100_{(2)}$

5) $110_{(2)} \cdot 11_{(2)}$

110

110

$10010_{(2)}$

6) $1101_{(2)} \cdot 101_{(2)}$

1101

0000

1101

$1000001_{(2)}$

7) $11011_{(2)} \cdot 1001_{(2)} = 11110011_{(2)}$

8) $110111_{(2)} \cdot 101101_{(2)} = 100110101011_{(2)}$

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1. 32.

+	0	1	2
0	0	1	2
1	0	2	10
2	2	10	11

·	0	1	2
0	0	0	0
1	0	1	2
2	0	2	11

$12\dot{0}1_{(3)} + 20\dot{1}2_{(3)} = 10220_{(3)}$

$1120221_{(3)} \cdot 2_{(3)} = 10011212_{(3)}$

$20012_{(3)} \cdot 12_{(3)}$

20012

110101

$1010221_{(3)}$

1.33. a) baza 4

+	0	1	2	3
0	0	1	2	3
1	1	2	3	10
2	2	3	10	11
3	3	10	11	12

·	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	10	12
3	0	3	12	21

b) baza 5

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

·	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

c) baza 6

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	10
2	2	3	4	5	10	11
3	3	4	5	10	11	12
4	4	5	10	11	12	13
5	5	10	11	12	13	14

·	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	10	12	14
3	0	3	10	13	20	23
4	0	4	12	20	24	32
5	0	5	14	23	32	41

1. 34. 1) $11.1_{(2)} = 1 \cdot 2 + 1 + 1 \cdot 2^{-1} = 3.5$

2) $101.101_{(2)} = 1 \cdot 2^2 + 1 + 1 \cdot 2^{-1} + 1 \cdot 2^{-3} = 5 + \frac{1}{2} + \frac{1}{8} = 5 + 0.5 + 0.125 = 5.625$

3) $111.111_{(2)} = 1 \cdot 2^2 + 1 \cdot 2^1 + 1 + 1 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3} = 7 + 0.5 + 0.25 + 0.125 = 7.875$

4) $1000.0001_{(2)} = 1 \cdot 2^3 + 1 \cdot 2^{-4} = 8 + \frac{1}{16} = 8 + 0{,}0625 = 8.0625$

1.35. 1) $\frac{1}{2} = 0.1_{(2)}$

2) $\frac{1}{8} = \frac{1}{2^3} = 1 \cdot 2^{-3} = 0.001$

3) $\frac{3}{2} = 1 + \frac{1}{2} = 1 + 0.1 = 1.1_{(2)}$

4) $\frac{1}{2^n} = 0.\underbrace{00\ldots0}_{n-1\ \text{nula}}1$

5) $0.25 = \frac{1}{4} = \frac{1}{2^2} = 0.01_{(2)}$

6) $2.5 = 2 + 0.5 = 2 + \frac{1}{2} = 10 + 0.1 = 10.1_{(2)}$

7) $12.125 = 12 + 0.125 = 1100 + 0.001 = 1100.001_{(2)}$

8) $2.0625 = 2 + 0.0625 = 2 + \frac{1}{16} = 2 + \frac{1}{2^4} = 10 + 0.0001 = 10.0001$

1.36. 1) $11.11_{(2)} + 100.1_{(2)} = 1000.01_{(2)}$

2) $1.101_{(2)} + 11.01_{(2)} = 100.111_{(2)}$

3) $110.011_{(2)} - 10.110_{(2)} = 11.101_{(2)}$

4) $11011.111_{(2)} - 10.1011_{(2)} = 11001.0011_{(2)}$

5) $11.1_{(2)} \cdot 1.01_{(2)} = 100.011_{(2)}$

6) $1.101_{(2)} \cdot 1.01_{(2)}$

$$\begin{array}{r} 1101 \\ 0000 \\ \underline{1101} \\ 10.00001 \end{array}$$

Sami brojeve prevedite u dekadski sustav te provjerite rezultate.